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A Compact Review of Multi-criteria Decision Analysis Uncertainty Techniques

by John W. Gerdes and Eric Spero

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14. ABSTRACT

Multiple criteria decision analysis is widely used in a variety of fields, and offers a normative technique to justification of a decision. In most real decisions, uncertainty will be present, and the techniques for dealing with uncertainty are varied and depend on the properties of the decision to be made. This work summarizes a variety of relevant techniques and provides some commentary about their history, intended applications, strengths, and weaknesses. This work is intended to introduce the techniques and provide the novice with an overview of some available methods. For each of the techniques presented, a wealth of sources are cited for more detailed study so that a chosen method may be applied to a decision of interest.

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1. Introduction

1.1 Purpose

The purpose of this work is to survey the literature in multi-criteria decision analysis (MCDA) techniques to provide an overview of the field. A particular focus is on methods for dealing with uncertainty or incomplete information in the decision process. The survey seeks to provide an introduction to some of the most popular analysis tools and techniques available to decision makers, and some insight into when each tool is appropriate to use and how the tools may be used in a structured analysis framework to solve complex problems when faced with uncertainty.

1.2 Motivation

In the 21st century, decision makers in a variety of fields are faced with increasingly challenging decisions. Challenges to the decision maker include multiple objectives with value tradeoffs, difficulty in identifying good alternatives, the desire for robust decisions when faced with long time horizons, multiple impacted groups, risk and uncertainty, interdisciplinary challenges, and others (*1*). These challenges necessitate a structured approach to decision analysis that enables sound decisions in a reasonable amount of time. To address the needs of decision makers, decision analysis techniques have developed. In general, *"Decision analysis provides tools for quantitatively analyzing decisions with uncertainty and/or multiple conflicting objectives"* (*2*).

Central to this definition is the notion of multiple conflicting objectives. In most real-world cases, a decision will not have a simple "best" solution; therefore, the decision maker must seek a solution, set of solutions, or section of the solution space that satisfies multiple criteria in an optimal way, relative to their preferences. Therefore it is clear that establishing the preference structure of the decision maker is an important step in the decision process. Preference models capture the decision maker's risk attitudes, behaviors, and requirements, enabling better decisions with regard to the realized utility of a decision.

MCDA in real-world problems requires consideration of a few key sources of non-ideal information to obtain a useful result (*3, 4*). Imprecise, uncertain, and ill-defined features of decisions and the context in which the decision is made may affect the decision. Furthermore, the solution may be unstable with regard to the system of values to be used to describe preferences relating to the decision. The decision analysis technique needs to account for these challenges, and a variety of approaches have been proposed (*5–7*). These may be broadly categorized into multi-valued logic (*10, 11*), fuzzy sets (*16–20*), utility theory (*1, 8, 9*), rough set theory (*21*), and discrimination thresholds or quasi or pseudo-criterion (*12–15*) used in outranking methods.

2. Preference Modeling with Uncertainty in Support of MCDA

Preference modeling is a key step in the decision analysis process involving the classification of alternatives into a preference structure based on some data. A wide range of fields make use of preference modeling including psychology, medicine, economics, programming, biology, politics, engineering, and many others. Depending on the desired type of classification a variety of preference structures are available (*22*).

Preference modeling can present challenges to the decision maker because it is frequently based on the translation of abstract statements from the decision maker. When the decision maker does not have a definite preference structure or understanding of why or how to rank two alternatives, preference modeling techniques are required to deal with the incomplete information (*23–25*).

Methods to obtain and validate preference information are important to consider and contribute significantly to the quality of the techniques used in MCDA. Additional information about preferences and any ambiguities may also enable a deeper understanding of the decision. In most practical decision settings, there will be some uncertainty in the preferences of the decision maker; therefore, a variety of tools and techniques have emerged to account for this in a structured way.

2.1 Multi-valued Logics

Traditional logic is reliant on a binary truth value, 0 or 1. This truth value may be assigned to build a preference relationship, and there is plenty of theory on this approach (*26, 27*). In many cases, it is not possible to establish the exact relationship of objects under comparison. This problem may arise when there are degrees of truth to a given statement, requiring the use of more than the traditional two truth values. Additionally, sometimes a decision maker will seek a greater understanding of not only the decision to be made but also the state of information leading to a decision. Four-valued logics are a formal basis for describing the usual true and false states of Boolean logic, as well as describing the state of information as known or unknown. The logical connectives used in four-valued logic are defined as "∨" (or), "∧" (and), "~" (complementation), " ≈" (semi-negation), and "¬' (negation). Table 1 shows the truth tables associated with these connectives where t, k, u, and f correspond to true, known, unknown, and false, respectively.

Table 1. Truth tables of ¬, ~, ≈, and combinations (*10*).

α	$\approx\alpha$	$\sim\approx\alpha$	$\sim\alpha$	$\neg\alpha$	$\neg\approx\alpha$	$\neg\sim\approx\alpha$	$\neg\sim\alpha$
t	*k*	*u*	*f*	*f*	*k*	*u*	*t*
k	*t*	*f*	*u*	*k*	*f*	*t*	*u*
u	*f*	*t*	*k*	*u*	*t*	*f*	*k*
f	*u*	*k*	*t*	*t*	*u*	*k*	*f*

Practically, the purpose of the negation operator is as expected from traditional logic, the complementation operator allows for the expression of inconsistency, and the semi-negation operator allows for the expression of possibility.

First introduced in the literature in (*28, 29*), four-valued logic enables the formal characterization of different states of hesitation when preferences are modeled (*10, 30*). These states of hesitation may result from degrees of truth or some combination of either incomplete or contradictory information. The usage of four-valued logic to model preferences was suggested in references 31 and 32. In more complex decisions, independence of criteria is almost never achievable, which necessitates the aggregation of interacting criteria (*33*).

2.2 Fuzzy Sets

Fuzzy sets were introduced by Dr. Lotfi Zadeh of University of California (UC) Berkeley (*34, 35*). In real-world decisions, they serve a variety of useful purposes for the representation of decisions. The theory is based on the idea of moving from binary to continuous representations of alternatives. A fuzzy set F on a set Ω is defined by the application of a membership function μ_F, shown in equation 1, the definition of a fuzzy set (*12*):

$$\mu_F : \Omega \rightarrow [0,1] \tag{1}$$

The purpose of fuzzy sets is often to represent imprecisely known concepts or vaguely perceived concepts as linguistic variables (*12*). As such, members of a fuzzy set have varying grades of membership or varying levels of truth, corresponding to the parameters with which the decision maker is presented. In this regard, fuzzy sets may be thought of as a technique that enables computers to deal with uncertainty as the human mind does. Humans are capable of considering the vagueness and uncertainty associated with a problem, and by using fuzzy sets, computers may be programmed to perform similarly. In the context of a MCDA, fuzzy sets may be used either to represent criteria values of alternatives or preference relations.

As a simple example, say a person asks which persons from a selected population are considered to be tall. The tallness criterion is inherently fuzzy, as it depends on a variety of unknown

factors including the relative height of the subject, the average height of some population, and perhaps other extraneous factors. Clearly in this case, a binary description of tall does not provide much information. Therefore, the decision maker must construct a membership function, which enables the assignment of tallness to each member of the selected population. This process forces the decision maker to think carefully about how to map the linguistic variable of tallness into a clearly defined mathematical function that captures more information than a binary representation. The theory of fuzzy sets has been widely researched, enabling decision makers to make use of this technique to support multiple criteria decision making (MCDM) (*36–43*). Topics include how to obtain and validate preference information (*44*, *45*) and the relationship between preferences and the value system of the decision maker (*46–49*). In outranking decisions, fuzzy sets have been used in combination with analytical hierarchy process (AHP) and technique for order preference by similarity to ideal solution (TOPSIS), and are described in some more detail in section 3.

2.3 Utility Theory

Utility may be thought of as a metric that captures the usefulness, desirability, or benefit of a good or service under consideration. Utility theory, a normative decision technique, is used by the decision maker to pick the alternative that will provide the greatest expected utility. Utility theory is a valuable tool because it provides a logical basis for decision makers when certainty is not guaranteed. The description of preferences is frequently accomplished with the aid of utility theory. Utility theory is a wide-ranging field, but the seminal work in this area is without question Keeney and Raiffa's 1976 book *Decisions with Multiple Objectives* (*8*).

Modeling the preferences of a decision maker that is presented with a set of alternatives described by several points of view is an important process in exercising a normative decision theory. The available alternatives are characterized by a number of criteria, and the ultimate goal of the decision maker is to decide which alternative is most preferred. For each alternative, the criteria are input into some function that maps criteria values to utility values. The functional representation of a decision maker's preferences is dependent on a variety of factors that describe the decision and decision maker. One key concept in defining a utility function is the decision maker's risk attitude. In order to establish an accurate functional representation of preferences, risk must be accounted for. Sometimes, a representation of preferences where certainty is assured is called a value function, where a similar functional model of preferences accounting for risk is called a utility function. The concept of expected utility theory describes the risky decision (*50, 51*). An interesting note about expected utility is that while it may aid the decision maker in selecting the most desirable alternative, in reality, the expected utility may not end up providing the greatest utility. A simple example would be in a game of blackjack where a player makes a decision to "hit" in a non-ideal scenario. The player might still end up winning the

hand, even though a normative theory of the game would dictate that the best course of action or the alternative with the highest expected utility was to "stay."

The construction of utility functions can be quite challenging, as it is unnatural for a person to simply state the nature of their preferences in a mathematical function. Furthermore, complications like interactions between criteria make the construction of an accurate utility function even more challenging. To address this problem while ensuring inter-criteria compatibility, some generalized techniques exist to enable the construction of a function (*33*), as well as literature describing the underlying mathematics (*52–54*), which draw from fuzzy set theory.

3. MCDM Outranking Techniques

In a wide range of problems, the decision maker is seeking the best alternative, and the outranking methods provide a structured and normative approach. In this section, some of the most popular methods are described, with some emphasis on how they deal with uncertainty in the decision, an unavoidable reality of real-world decisions.

3.1 Dominance-based Rough Set Approach

The Dominance-based Rough Set Approach (DRSA) to MCDA is based on rough set analysis, which seeks to explain the dependence or relationship between the values of some decision attributes (dependent) and other condition attributes (independent) (*55, 56*). In this regard, the technique is similar to data mining and artificial intelligence techniques (*57*). Rough set theory is a useful tool for analysis of vaguely described decision situations (*58, 59*). The procedure involves constructing a table that contains the alternatives or actions as its rows and the criteria that are rated as its columns. The table is then populated and searched for insightful information that is used to create decision rules. An example of such a decision situation has been composed in table 2.

Table 2. Data table for student comprehensive evaluations, adapted from reference 21.

Student	Math	Physics	Literature	Comprehensive
S1	Good	Medium	Bad	Bad
S2	Medium	Medium	Bad	Medium
S3	Medium	Medium	Medium	Medium
S4	Good	Good	Medium	Good

DRSA requires a data table of this form, which consists of objects or actions along the rows, attributes along the columns, and evaluations that are contained in each cell of the table. The attributes may be divided into two distinct groups. The first group, denoted C, contains

condition attributes. In table 2, the condition attributes are Math, Physics, and Literature. The second group, denoted D, contains decision attributes, in this case the comprehensive evaluations. DRSA seeks to explain the evaluations in D using the evaluations in C. Therefore, the case where there are multiple attributes in D corresponds to the existence of multiple decision makers.

The dominance principle, which is based on axioms of logic, is used to analyze the results of the evaluations to obtain useful data about the decision maker(s). The dominance principle simply states that if alternative *a* is better than alternative *b* on all criteria, *a* is better than *b*. By searching for evaluations on criteria that always obey the dominance principle, one can determine where the decision maker is certain and where a hesitation (inconsistency) has occurred, and thus generate a decision rule that captures this information. Such information is one of the interesting features of DRSA analyses. In this case, it can be seen that there is a hesitation in the ranking of students S1 and S2, since S1 dominates S2 in the condition attributes, yet not in the decision attribute. While the example is somewhat trivial here, in more complex decisions, such insights are useful in describing a preference model for the decision maker, and making recommendations for future decisions.

Typically, DRSA is most applicable to sorting and classification type decisions; however, it may also be extended to choice and ranking type problems by using pairwise comparisons between alternatives instead of comparing each alternative to a fixed measurement threshold or set of categories. In the literature, the relative importance and interactions of the decision criteria have been analyzed (*60*, *61*). A variable consistency model of the DRSA allows for flexibility in the assignment of objects to decision classes (*62*). Extensions of the DRSA include induction of decision trees (*63*), and joint consideration of dominance, indiscernibility, and similarity relations, which is particularly useful in practical data mining applications (*64*). For a complete treatment of DRSA and some additional extensions including techniques for dealing with fuzzy dominance relations and incomplete data tables, see reference 21.

3.2 Data Envelopment Analysis

"Data Envelopment Analysis is a non-parametric method for evaluating the relative efficiency of decision-making units (DMUs) on the basis of multiple inputs and outputs" (*65*). Data Envelopment Analysis (DEA) was first described by Charnes, Cooper, and Rhodes (*66*) and later extended by Banker, Charnes, and Cooper (*67*). In general, DEA tends to be used in cases where a focus on efficiency in terms of return on invested resources is of interest to the decision maker.

An example decision situation is shown in table 3. For an illustrative example, the alternatives may be thought of as retail outlets. Each store will have inputs such as stock, payroll, bills, and other expenses, which ideally would be minimized. There will also be outputs such as sales or profits that should be maximized.

Table 3. Comparison of alternatives on input and output flows adapted from reference 68.

	Input 1 (v_1)	**Input 2 (v_2)**	**Output 1 (u_1)**	**Output 2 (u_2)**
Alternative 1 (j_1)	10	10	10	10
Alternative 2 (j_2)	9	11	12	8
Alternative 3 (j_3)	2	3	4	1
Alternative 4 (j_4)	20	30	1	30

The usual measure of efficiency would be to generate the largest possible ratio of outputs to inputs, the Pareto frontier. In a simplified case where there is only one input and two outputs, it is simple to visualize an efficient frontier. This is shown graphically in figure 1, where P' indicates the targets for the inefficient alternative P.

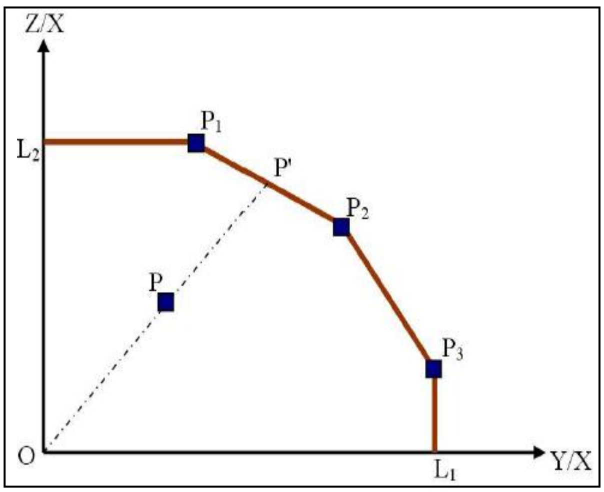

Figure 1. Efficient frontier for input X and outputs Y and Z (*69*).

It is difficult to visualize and make definitive statements about the efficiency of the alternatives shown in table 3, due to the presence of multiple inputs and outputs. In many real-world decisions, the problem becomes intractable using this technique of visualizing the efficient frontier, due to the challenge of comparing arbitrarily large numbers of inputs and outputs.

One way to determine efficiency is to use the ratio of a weighted sum of outputs to a weighted sum of inputs; however, this approach is unsuitable when the weights are difficult to calculate due to uncertainty or if the inputs and outputs are incommensurate, thus making a relative weighting challenging.

Therefore, DEA takes a different approach. The efficiency of an alternative is calculated by adopting a set of weights that shows each alternative in the most favorable way in comparison to the other units. This is set up in equation 2 (measure for relative efficiency of unit j), where the efficiency ε is calculated as the ratio of a weighted sum of outputs to a weighted sum of inputs.

$$\varepsilon = \frac{u_1 y_{1j} + u_2 y_{2j} + \dots}{v_1 x_{1j} + v_2 x_{2j} + \dots} \tag{2}$$

where u_1 = weight given to output 1

y_1 = amount of output 1 from unit j

v_1 = weight given to input 1

x_{1j} = amount of input 1 to unit j

The efficiency of unit j_0 may then be calculated as the solution to the linear program in equation 3 (the DEA framwork)

$$\boldsymbol{Max\ h_0} = \frac{\sum_r \boldsymbol{u_r y_{rj_0}}}{\sum_i \boldsymbol{v_i x_{ij_0}}} \tag{3}$$

$$subject\ to\ \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}}\ for\ each\ alternative\ j\ with\ u_r, v_i\ \geq \delta$$

The purpose of forcing u_r and v_i to be at a minimum some small positive quantity δ is to ensure no input or output is completely ignored in the determination of efficiency. Put in practical terms, equation 3 seeks to maximize the efficiency of unit j_0 subject to the efficiency of all units being ≤1. The solution will produce the weights most favorable to unit j_0 and serve a measure of efficiency. The key benefit to this technique is the identification of the weights most favorable to a particular unit under examination. As there will often be cases where units value their inputs or outputs differently, this helps to fairly compare each unit in determining the overall most efficient unit. A simple example would be a comparison of schools, where one school values achievement in sports more than another school that values achievement in music.

Applying this technique to alternative 1 yields equation 4, which is the application of DEA Framework to table 3:

$$Max\ h_0 = \frac{10u_1 + 10u_2}{10v_1 + 10v_2} \tag{4}$$

$$subject\ to\ \frac{12u_1 + 8u_2}{9v_1 + 11v_2} \leq 1\ (Alternative\ j1)$$

$$\frac{4u_1 + 1u_2}{2v_1 + 3v_2} \leq 1\ (Alternative\ j2)\ \dots for\ remaining\ alternatives\ and\ u_1, u_2, v_1, v_2 \geq \delta$$

The solution to the system of equations gives a value h0 for the efficiency of alternative 1. If this value is equal to 1, alternative 1 is the most efficient. If it is less than 1, some other alternative is more efficient. The solution of the linear program provides a variety of useful information including the most efficient alternatives, and a set of target inputs and outputs for inefficient alternatives.

In real-world problems, input and output data will frequently be imprecise, for a variety of reasons. DEA has been extended by Cooper et al. (*70*) to the case where crisp values are not

available through imprecise data envelopment analysis (IDEA). IDEA uses one of four approaches to deal with uncertainty in a decision. The tolerance approach, the a-level based approach, the fuzzy ranking approach, and the possibility approach. Each of these four methods is widely examined in the literature (*68, 69*).

A very thorough discussion of DEA is offered in reference 68 and an exhaustive literature search (*71*) describes some trends in publications and usage of the technique. In addition, a literature review focused on IDEA is available, focusing on techniques for dealing with uncertainty (*72*). The IDEA technique is widely used in banking, education, health care, and hospital efficiency. Other topics include production, armed forces, sports, market research, highway maintenance, courts, and benchmarking.

3.3 ELECTRE Method

ELECTRE is an acronym that stands for Elimination Et Choix Traduisant la Realité (Elimination and Choice Expressing the Reality) (*73*). ELECTRE spawned as a way to deal with the shortcomings of an existing multiple criteria weighted-sum technique called the Method of Analysis, Research, and Selection of New Activities (MARSAN) (*74*), which was in use at the European company SEMA. A detailed history of the origins of ELECTRE may be found in reference 75, and a description of the wide applicability to different fields may be found in reference 76.

ELECTRE methods are intended for decision situations where the decision maker wants to include at least three criteria in the model, plus at least one of the following properties (*74, 77, 78*):

1. Comparison of differences is difficult or artificial because actions are evaluated on an ordinal scale or a weakly interval scale.
2. The nature of evaluations is strongly heterogeneous among criteria, making it difficult to aggregate all the criteria in a common scale.
3. Balancing of losses on one criterion by gains in another criterion may be unacceptable, requiring noncompensatory aggregation procedures.
4. Small differences of evaluations are not significant in terms of preferences, while the accumulation of several small differences may become significant.

ELECTRE models actions or alternatives through outranking relations structured in one of three ways. Strict preference, expressed as aSb, means a is strictly preferred to b. Indifference, expressed aIb, means a and b are similar. Incomparability, expressed aRb, is equivalent to not aSb and not bSa simultaneously. The preference relations built may be crisp or fuzzy.

Once preferences are modeled, ELECTRE methods proceed with two major phases. First, outranking relations are constructed. Second, recommendations from the results of the outranking relations are generated using an exploitation procedure. Choice, ranking, and sorting problems each have unique exploitation procedures (*77*, *79–82*).

ELECTRE I is the most simplified method and is only to be used when criteria are using identical numerical scales with identical ranges. The criteria are described as a set J and w_j is the weight of the jth criterion, with the sum defined as 1 for simplicity. The method defines strong concordance as a weight of [0.5,1].

$$\sum_{j \epsilon J} w_j = 1 \tag{5}$$

The concordance index supporting the claim that aSb is then defined in equation 6, where $\{j{:}g_j(a)>g_j(b)\}$ is the set of indices for all the criteria g_j supporting the outranking relation aSb, also called the concordant coalition.

$$c(aSb) = \sum_{\{j:\, g_j(a) \geq g_j(b)\}} w_j \tag{6}$$

In addition to the concordance index, discordance against "a is at least as good as b", defined in equation 7, must remain below a level defined by the decision maker. In equation 7, $j{:}g_j(a)<g_j(b)$ are the indices of the discordant coalition.

$$d(aSb) = \underset{\{j: g_j(a)<g_j(b)\}}{max} \{g_j(b) - g_j(a)\} \tag{7}$$

Of greater interest is the ELECTRE III method, designed to deal with inaccurate, imprecise, uncertain, and ill-defined data. The key difference incorporated into ELECTRE III is the usage of fuzzy relations to describe the outranking relation. The credibility index, denoted p(aSb), is defined using the concordance index c(aSb) and the discordance index $d_j(aSb)$ for each criterion g_j, which is proportional to the difference $g_j(b)–g_j(a)$ as described in equation 8. In addition, a veto condition is described with two parameters v_j and p_j, described in greater detail in reference 74.

$$d_j(aSb) = \begin{cases} 1 \; if \; g_j(b) > g_j(a) + v_j(g_j(a)) \\ 0 \; if \; g_j(b) \leq g_j(a) + p_j(g_j(a)) \\ \frac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j\left(g_j(a)\right) - p_j(g_j(a))} \; otherwise \end{cases} \tag{8}$$

The credibility index p(aSb) is then defined as equation 9.

$$p(aSb) = c(aSb) \prod_{\{j \in J:\, d_j(aSb) > c(aSb)\}} \frac{1 - d_j(aSb)}{1 - c(aSb)} \tag{9}$$

The credibility index will then be the comprehensive concordance index when there are no discordant criteria and zero when a discordant criterion activates its veto power. For the case in

which the concordance is lower than the discordance on some discordant criterion, the credibility index is lower than the concordance because of the opposition effect.

At this point, the exploitation procedure proceeds by way of the ELECTRE I algorithm. The algorithm distills the best alternatives through successive partitioning and pre-ordering steps, until the final ordering is obtained (*74*).

3.4 PROMETHEE-GAIA Method

Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) and Geometrical Analysis for Interactive Aid (GAIA) were developed by Professors Jean-Pierre Brans and Bertrand Mareschal in 1982 (*83, 84*). In the development of the method, the authors sought to account for the following:

1. deviations between the evaluations of the alternatives within each criterion,
2. scaling effects due to disparate unit systems,
3. in pairwise comparisons, provide information about preference, indifference, or incomparability,
4. avoid black box procedures or parameters that confound the understanding of the method by the decision maker,
5. provide information on the conflicting nature of the criteria, and
6. provide sensitivity tools to test the effect of variable weights on criteria.

The last item is of special interest due to the difficulty of accurately estimating the relative weights of the criteria. With a sensitivity analysis capability, the decision maker can have a greater understanding of the importance of their selections.

The PROMETHEE method was designed to perform MCDA while accounting for each of these requirements, and consists of information between the criteria and information within each criterion. These correspond to weights of relative importance of the different criteria and a number of preference functions that describe the nature of preference as it relates to the deviation between evaluations of two alternatives on a particular criterion, respectively. The weights of relative importance of criteria are straightforward and represent a degree of freedom to the decision maker to explore the sensitivity of the decision to differing weight allocations. The preference functions enable the decision maker to capture the many ways that an evaluation might be scored. For example, in some cases, there may be a threshold below which deviation is considered negligible, followed by a range of deviation values for which increasing preference exists. Such a preference function is called a V-shape with indifference criterion. There are also functions to describe Gaussian preference distributions, step functions, binary functions, and

multi-level functions. The decision maker is free to choose the preference function that accurately describes the preferences they have given the set of alternatives under evaluation.

Once the weights have been assigned and the preferences assigned, the PROMETHEE procedure is applied by summing aggregated preference indices over all the criteria as in equation 10.

$$\begin{cases} \pi(a,b) = \sum_{j=1}^{k} P_j(a,b)w_j \\ \pi(b,a) = \sum_{j=1}^{k} P_j(b,a)w_j \end{cases} \tag{10}$$

The preference indices are expressing the degree to which a is preferred to b, and the degree to which b is preferred to a, over each criterion j, with weight w. Next, the positive and negative outranking flows are calculated as equations 11 and 12, respectively.

$$\phi^{+}(a) = \frac{1}{n-1} \sum_{x \epsilon A} \pi(a,x) \tag{11}$$

$$\phi^{-}(a) = \frac{1}{n-1} \sum_{x \epsilon A} \pi(x,a) \tag{12}$$

A variety of information follows from this computation. Degrees of preference, indifference, and incomparability are explored by examining the consistency of values calculated. In addition, the complete ranking is obtained by summing the two outranking flows. More details on this procedure are available in literature about the usage of PROMETHEE (*85, 86*).

The GAIA visual interactive module is a tool that enables visualization of the n alternative by k criteria decision space. By defining a plane as the two eigenvectors (or a volume as three eigenvectors) corresponding to the largest eigenvalues of the covariance matrix of the single criterion net flows, as much information as possible is preserved during the projection procedure. The decision maker may then explore graphically how each alternative performs in these alternatives in a simple visual depiction. A more detailed description of the usage of the GAIA module is available (*87, 88*).

PROMETHEE has extensions that enable MCDA under constraints (*89*), fuzzy MCDA (*90*), group decision support (*91, 92*), and sensitivity analysis (*93*). Also available is a PROMETHEE-GAIA software tool called Visual PROMETHEE (*94*). The Visual PROMETHEE tool is a comprehensive MCDA package based on the PROMETHEE approach that uses a variety of visual representations of the decision problem to aid the decision maker. A thorough instruction manual is available (*94*) that offers a useful and practical introduction to the technique and its implementation in real decisions.

3.5 Analytic Hierarchy Process

AHP is a theory used to relatively measure both tangible and intangible criteria on an absolute scale based on expert knowledge and existing measurements. The decision problem is structured

in a hierarchy with the goal at the highest level, the criteria on the second level, and the alternatives to be ranked on the bottom level. A notional hierarchy is shown in figure 2.

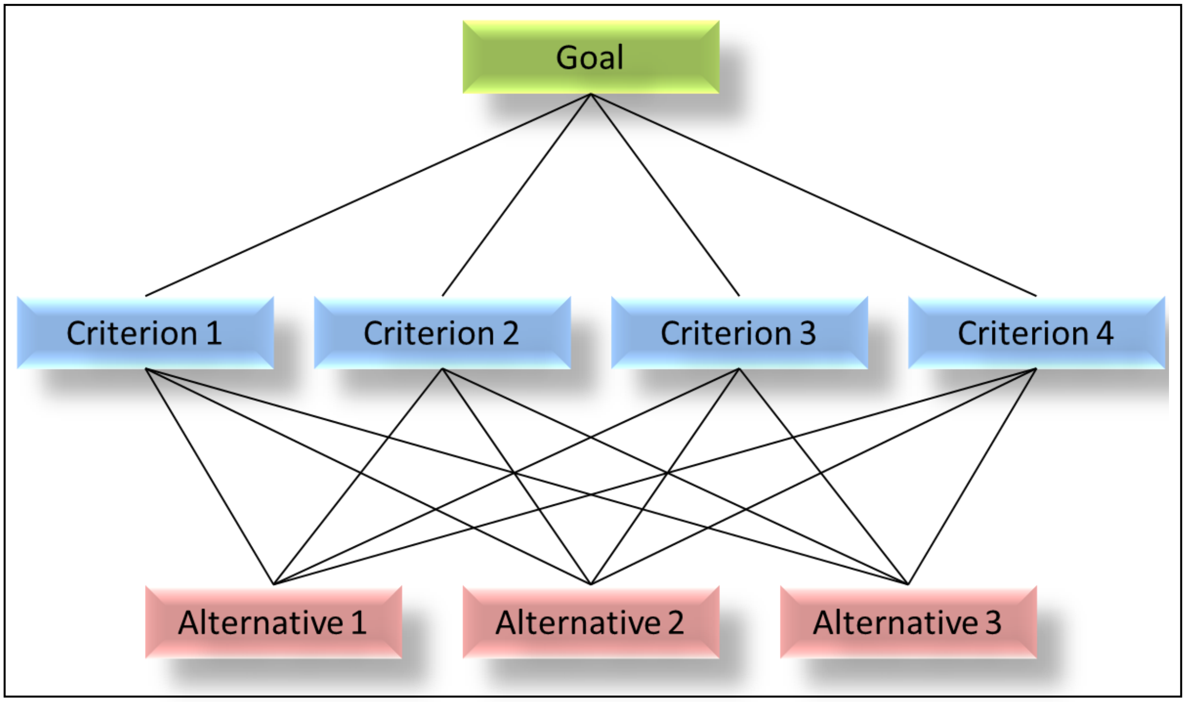

Figure 2. AHP notional hierarchy.

The connections between each level of the hierarchy represent a rating with respect to the level above, so it follows that each criterion is weighted in terms of its importance in achieving the goal, and subsequently, each alternative is rated on its ability to satisfy a given criterion, providing a ranking of alternatives. This framework of evaluation that is the foundation of AHP may be extended to multiple levels if the problem is too complex to express in this simple three level format. A key limitation of AHP is the relationships are expressed as linear weighted sums; therefore, nonlinear relationships may be lost in trying to compute the best alternative. This will become clearer as the evaluation is explored in more detail.

The first step is to look at the top level of the hierarchy and determine how much each criterion matters in accomplishing the goal by determining weighting factors. In the criterion weighting step, a series of pairwise comparisons are made between each of the criteria. For a pair under comparison, a notional matrix is constructed as shown in table 4 with the rating of a_{nm} indicating the relative importance of the criterion in row n to column m.

Table 4. Criteria pairwise comparison.

	Criterion 1	Criterion 2	Criterion 3
Criterion 1	1	a_{12}	a_{13}
Criterion 2	$1/a_{12}$	1	a_{23}
Criterion 3	$1/a_{13}$	$1/a_{23}$	1

The decision maker typically uses the scale in table 5 to describe the degree of increased importance from the activity in a row relative to an activity in a column. Reciprocals of this scale indicate the same importance degree, but in the opposite direction, e.g., 9 indicates that element 1 is extremely more important than element 2 while 1/9 denotes that element 1 is extremely less important than element 2.

Table 5. AHP rating table.

Rating	Description
1	Equal
3	Experience and judgment slightly favor one activity over the other
5	Experience and judgment strongly favor one activity over the other
7	Activity is very strongly favored over another, dominance demonstrated in practice
9	Highest possible dominance of one activity over another

The next step is to calculate the weights for each criterion. This is done by computing the principal eigenvector of the comparison matrix. The normalized values in this eigenvector will simply be the weights that correspond to the decision maker's pairwise comparisons. Thus, by setting up a simple eigenvalue problem from the pairwise comparison matrix, the decision maker has assigned weights to all the criteria using a structured approach.

The next step is to check the quality of these weights. The quality of weights that come from this analysis is referred to as consistency. Thus, ensuring consistency is analogous to ensuring the decision maker has not made circular choices (i.e., rock-paper-scissors). By defining a consistency index as equation 13, the level of consistency may be quantified.

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{13}$$

In equation 13, λ_{max} is the maximum eigenvalue and n is the size of the matrix. The consistency index is a somewhat abstract value without a point of reference. Therefore, it is compared to a random consistency index. By calculating the average consistency index of many randomly generated matrices of the same size, a point of comparison is generated to put the consistency index in perspective (*95*). The consistency ratio (CR) is then defined as the ratio of the random consistency index to the consistency index generated by the decision maker's pairwise comparison matrix, shown in equation 14, where n is sufficiently large enough for CR to converge.

$$CR = \frac{CI}{\sum_{i=1}^{n} CI_{random}} \tag{14}$$

A totally consistent set of judgments will yield CR=0, but in practical situations a larger value is typically expected. It is up to the judgment of the decision maker to set an acceptable value on the consistency ratio, as it may vary depending on the problem. A CR<0.1 is recommended by Saaty (*96*). The feedback provided to the decision maker during this analysis is one of the benefits of AHP, as it helps to identify areas where assumptions may have been inaccurate, and quantifies the degree of certainty in the weighting of criteria.

AHP can be used in a group setting to make decisions by collecting judgments from many respondents simultaneously and aggregating the results. Fuzzy decision-making techniques offer some flexibility in reconciling the differences of individual decision makers (*97*). The Fuzzy Analytic Hierarchy Process (FAHP) technique applies fuzzy numbers to any uncertain, ill-defined, or linguistic criteria. Sub-totals are then calculated for each row of the pairwise comparison matrix and a set of lower, middle, and upper possibilities is obtained. More detailed descriptions of the possible techniques for conducting the outranking based on this information are available as well as some applications for the usage of FAHP (*98–100*).

In the case of decisions that are not well suited to a hierarchical decomposition, the analytic network process (ANP) was devised, which generalizes the AHP to the case where high-level elements interact and depend on low-level elements in the traditional AHP hierarchy. ANP may also deal with fuzziness of the decision (*101*). The author of these techniques, Thomas Saaty, has published some popular references that thoroughly discuss the usage of these techniques (*102, 103*). The software tool Super Decisions, developed by Saaty, is available for download and is based on ANP (*104*). In addition, another software package based on AHP is available from Decision Lens (*105*).

3.6 Technique for Order Preference by Similarity to Ideal Solution

TOPSIS was first proposed by Hwang and Yoon in 1981 (*106*). The technique attempts to rank alternatives by their nearness to the ideal solution and their farness from the negative ideal solution, the identification of which may depend on some subjective judgments from the decision maker. The positive ideal solution is generated by maximizing benefit criteria while minimizing cost criteria. Similarly the negative ideal solution minimizes benefit and maximizes cost. Therefore, the positive ideal solution contains all the best possible values for all criteria. TOPSIS is implemented by first composing a normalized decision matrix, composed of each alternative and each criterion. Next, the positive and negative ideal solutions are determined. Third, the distance of each alternative to both of the ideal solutions is calculated. For each alternative, each criterion may be thought of as a dimension in a vector. The distance is calculated by summing the absolute value of the each criterion’s distance to the ideal solution. With these distances, the closeness coefficient of each alternative is calculated as the negative

ideal distance divided by the sum of the positive and negative ideal distances. This value then provides the decision maker with a ranking of the alternatives (*107*).

A large body of research has emerged that uses FAHP together with TOPSIS for MCDA (*108–114*). In general, FAHP is used to prioritize the criteria, and assign weights, while accounting for uncertainty. Subsequently, TOPSIS is used to rank the alternatives. Typically, when FAHP is used to assign weights to criteria, the uncertainty of the decision maker is represented by assigning a triangular fuzzy number instead of a crisp number. This simply means that a weight of 1 is assigned at the perceived value, and as distance from that value increases, the weight linearly decays to zero with a slope chosen to represent the degree of uncertainty. The uncertainty is then propagated through the TOPSIS method and the best-ranked solution thus accounts for fuzzy numbers, which will almost always be present in a practical decision setting. By using both FAHP and TOPSIS together, decision makers may efficiently make a selection without an extraordinary expenditure of effort, as both of these methods are easy to implement.

3.7 Measuring Attractiveness by a Categorical Based Evaluation Technique

Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH) is a MCDA technique that uses qualitative judgments to quantify the preference for a set of alternatives. Typically, a variety of techniques (*33*, *4*, *115–117*) are used by the decision maker to quantify their ranking of the alternatives as well as the strength of their preference in relative terms. Such a task is not a simple undertaking, which led to the creation of the MACBETH method. The key motivation was to enable construction of a value scale without forcing the decision maker to try to assign numerical scores, which can be a challenging exercise.

The approach is similar to AHP in that it asks the decision maker to give a qualitative judgment of their preference in a series of pairwise comparisons with qualitative terms ranging from very weak to moderate to extreme. Once a matrix of these qualitative judgments has been populated, consistency criteria are checked to ensure the responses of the decision maker are acceptable. A linear algebraic approach represents the sets of inconsistent judgments and constraints and then uses an algorithmic approach to suggest changes to judgments or constraints that will resolve the inconsistencies. With the inconsistencies resolved, MACBETH then presents to the decision maker a visual representation of the preferred alternatives.

MACBETH has been used to derive preference scales in many applications including portfolio management, supplier performance evaluation, location of military facilities, and many others (*118–122*).

4. Conclusion

Uncertainty in MCDA is a challenging problem that demands a structured approach to properly model the decision maker. Fortunately, there exist a variety of approaches to the problem and a wide range of literature describing many problem types. This report has broadly discussed some concepts relating to preference modeling, outranking, and dealing with uncertainty in MCDA.

Whatever technique is used in MCDA, the decision maker may obtain additional information by conducting a sensitivity or robustness analysis. A variety of literature treating this analysis has been published in some useful areas. The goals of this analysis are to establish the appropriate method of problem solution (*123, 124*), arrive at robust solutions (*125–129*), and gain a deeper understanding about which portions of the problem are robust (*130, 131*).

The techniques discussed here were chosen because of their popularity in literature and practical usage; however there are many more techniques in existence. The reader is encouraged to search for additional techniques to supplement those listed here. Each decision problem has certain features that will be more effectively handled using particular MCDA techniques, therefore it is important to not only understand the problem, but also the methods available as well as their limitations. For a general summary of the challenges faced by decision makers and some techniques for formal modeling of uncertainty, references 5 and 132 are recommended for additional reading.

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List of Symbols, Abbreviations, and Acronyms

AHP	analytical hierarchy process
ANP	analytic network process
CR	consistency ratio
DEA	Data Envelopment Analysis
DMUS	decision-making units
DRSA	Dominance-based Rough Set Approach
ELECTRE	Elimination Et Choix Traduisant la Realité/Elimination and Choice Expressing the Reality
FAHP	Fuzzy Analytic Hierarchy Process
GAIA	Geometrical Analysis for Interactive Aid
IDEA	imprecise data envelopment analysis
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MARSAN	Method of Analysis, Research, and Selection of New Activities
MCDA	multi-criteria decision analysis
MCDM	multiple criteria decision making
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluation
TOPSIS	technique for order preference by similarity to ideal solution
UC	University of California

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